

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma (All)

Semester : 2

Date : 20/04/2019

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  - (2) Instructions written on main answer book are strictly to be obeyed.
  - (3) Draw neat diagrams and figures (if necessary) at right places.
  - (4) Assume suitable data if needed.
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**Q-1                      Attempt the following questions:                      (14)**

- a)  $\lim_{\theta \rightarrow 0} \frac{\tan \theta}{5\theta} = \underline{\hspace{2cm}}$   
(A) 1 (B) 0 (C) 1/5 (D) 5
- b)  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{x}{2}} = \underline{\hspace{2cm}}$   
(A)  $e^2$  (B)  $e$  (C)  $e^{1/2}$  (D) None of these
- c)  $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1} = \underline{\hspace{2cm}}$   
(A) -1 (B) 1 (C) 0 (D) 3
- d)  $\frac{d(7)}{dx} = \underline{\hspace{2cm}}$   
(A) -1 (B) 0 (C) 1 (D) None of these
- e)  $\frac{d(1/x)}{dx} = \underline{\hspace{2cm}}$   
(A)  $-\frac{1}{x^2}$  (B)  $\frac{1}{x}$  (C)  $e$  (D) 1
- f)  $\frac{d(e^{5x})}{dx} = \underline{\hspace{2cm}}$   
(A)  $5e^x$  (B)  $e^x$  (C)  $\frac{e^{5x}}{5}$  (D)  $5e^{5x}$
- g)  $\frac{d(\cot^{-1} x)}{dx} = \underline{\hspace{2cm}}$   
(A)  $\frac{-1}{\sqrt{1-x^2}}$  (B)  $\frac{1}{\sqrt{1-x^2}}$  (C)  $\frac{1}{1+x^2}$  (D)  $\frac{-1}{1+x^2}$



- h)  $\int 1 dx = \underline{\hspace{2cm}}$   
 (A) 0 (B) 1 (C)  $x+c$  (D) None of these
- i)  $\int a^x dx = \underline{\hspace{2cm}}$   
 (A)  $\frac{a^x}{\log_e a} + c$  (B)  $a^x \log_e a + c$  (C)  $a^x + c$  (D)  $\log_e a + c$
- j)  $\int_0^1 \frac{4}{1+x^2} dx = \underline{\hspace{2cm}}$   
 (A)  $\pi$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D) None of these
- k)  $|(2, 1, 1) + (1, 2, 3)| = \underline{\hspace{2cm}}$   
 (A) 10 (B)  $\sqrt{43}$  (C)  $\sqrt{34}$  (D) None of these
- l) If  $\bar{a} = i - j + k$  then  $\hat{a} = \underline{\hspace{2cm}}$   
 (A)  $\frac{(1, -1, 1)}{\sqrt{3}}$  (B)  $\frac{(-1, 1, 1)}{\sqrt{3}}$  (C)  $\frac{(1, 1, -1)}{\sqrt{3}}$  (D)  $\sqrt{3}$
- m) If  $\bar{x} = (1, 1, 1)$  and  $\bar{y} = (2, -1, 3)$  then  $\bar{x} \times \bar{y} = \underline{\hspace{2cm}}$   
 (A)  $(4, -1, 3)$  (B)  $(-4, -1, 3)$  (C)  $(-4, 1, 3)$  (D)  $(4, -1, -3)$
- n) If  $\bar{x} = (1, 1, 1)$  and  $\bar{x} = (2, -2, 1)$  then  $\bar{x} \cdot \bar{y} = \underline{\hspace{2cm}}$   
 (A) 5 (B) 1 (C) 0 (D) -1

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

- a) If  $\bar{a} = 3i - 2j + k$ ,  $\bar{b} = 2i - 4j - 3k$  and  $\bar{c} = -i + 2j + 2k$  (5)  
 then find modulus of  $2\bar{a} - 3\bar{b} - 5\bar{c}$ .
- b) Find unit vector which is perpendicular to both  $x = (3, 1, 2)$  and (5)  
 $y = (2, 1, 1)$ .
- c) Evaluate:  $\lim_{x \rightarrow 3} \frac{x^2 - x - 6}{x^3 - 3x^2 + x - 3}$  (4)

**Q-3 Attempt all questions (14)**

- a) Prove that  $\lim_{x \rightarrow a} \frac{\sqrt{2a-x} - \sqrt{x}}{a-x} = \frac{1}{\sqrt{a}}$ . (5)
- b) Evaluate:  $\lim_{x \rightarrow 0} \frac{x \tan x}{1 - \cos x}$  (5)
- c) Find  $\frac{dy}{dx}$  if  $y = \frac{a + b \sin x}{a \sin x + b}$ . (4)

**Q-4 Attempt all questions (14)**

- a)  $S = t^3 - 6t^2 + 8t - 4$  gives the distance travelled by a body in  $t$  seconds. Find (5)  
 velocity and acceleration at  $t = 4$  second.



b) Find  $\frac{dy}{dx}$  if  $y = \log\left(\frac{\sin x}{1 + \cos x}\right)$ . (5)

c) If  $\bar{a} = 2i - j$ ,  $\bar{b} = i + 3j - 2k$  then obtain  $\left|(\bar{a} + \bar{b}) \times (\bar{a} - \bar{b})\right|$ . (4)

**Q-5**

**Attempt all questions**

(14)

a) Prove that angle between two vectors  $3i + j + 2k$  and  $2i - 2j + 4k$  is (5)

$$\sin^{-1}\left(\frac{2}{\sqrt{7}}\right).$$

b) If  $y = 2e^{-3x} + 3e^{2x}$  then prove that  $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ . (5)

c) Evaluate:  $\lim_{n \rightarrow \infty} \frac{1^3 + 2^3 + \dots + n^3}{n^2(1 + 2 + \dots + n)}$  (4)

**Q-6**

**Attempt all questions**

(14)

a) A particle moves from the point  $-i - j - k$  to the point  $k + j + i$  under the effect of two constant forces  $2i + j + k$  and  $i + 3j + k$ . Find the work done. (5)

b) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$ . (5)

c) Find derivative of  $y = x^3 - 2x$  using first principle. (4)

**Q-7**

**Attempt all questions**

(14)

a) Evaluate:  $\int \frac{x^4 + x^2 + 1}{x^2 + 1} dx$  (5)

b) Find the volume of sphere of radius  $r$ . (5)

c) For what value of  $m$ , vectors  $mj + 2i + k$  and  $2i + 4j + 5k$  are perpendicular to each other? (4)

**Q-8**

**Attempt all questions**

(14)

a) Evaluate:  $\int x^n \log x dx$  (5)

b) Find  $\frac{dy}{dx}$  if  $y = (e^{3x} + 1)^{2x+5}$ . (5)

c) Evaluate:  $\int_0^{\frac{\pi}{3}} \frac{\sin x}{3 + 4 \cos x} dx$  (4)

